

# Rolling the *Black Pearl* Over: Analyzing the Physics of a Movie Clip

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In the third movie (“At World’s End”) in the *Pirates of the Caribbean* series, Jack Sparrow and his crew need to roll their ship (the *Black Pearl*) over in order to bring it back to the living world during a green flash at sunset. They do so by running back and forth from one side railing to the other on the top deck. In addition, Captain Barbossa orders that 18 cannons and a pile of barrels on the lower deck be cut loose to add mass to the running crew. In the movie they overturned the ship, but would they succeed under the same circumstances on a real galleon? In this paper, a numerical analysis using simple approximations is developed that suggests that what occurs in the movie is in fact realistic. Analyzing a popular film clip in this manner is a good way to arouse student interest, to teach about physics and numerical methods, and to model scientific reasoning when a situation is not as neatly defined as in a typical textbook problem.

Two important principles were used to develop the model presented here:

1. The analysis had to be kept at the introductory physics level, both so that it would be accessible to students and so that its plausibility would be firmly based on core concepts. Thoughtful readers will discover many points throughout the analysis where a more elaborate approach is possible. They are invited to extend the work, perhaps in the form of student projects.
2. The analysis had to use as few parameters as possible and their values had to be independently determined from external sources or from simple estimates. The more parameters in the model, the more one might legitimately question the robustness of the results. In addition, the analysis becomes overwhelming if one tries to account for every detail of the ship and its environment. An important goal of physics is to simplify a situation as far as reasonably possible.

## Theory of ship stability

Although a full-scale version of the *Black Pearl* was constructed for the second and third movies in the *Pirates of the Caribbean* series,<sup>1</sup> it is nevertheless a model and not a true tall ship. We therefore decided it would be more realistic to base the analysis on the characteristics of an authentic 17th-century Spanish galleon having the same general dimensions and appearance as the *Black Pearl*, namely the *San Juan Bautista*. According to Wikipedia,<sup>2</sup> some relevant parameters of this ship include:

- its displacement (total mass including cannons and crew) is  $M = 500$  long tons = 508 000 kg;

- its length is  $L = 55$  m (not including the front bowsprit);
- its beam (width at its widest point) is  $b = 11.3$  m;
- the draft (depth of the bottom of the keel from the water line) is  $d = 3.8$  m, labeled KO in Fig. 1(b).

From the movie itself, we count 16 crew members running from side to side on the top deck, each of which we take to have a typical mass (including clothing and gear) of 88 kg. The lower deck carries 18 twelve-pound cannons,<sup>1</sup> which have a mass (including their wheeled carriage) of about 2 metric tons = 2000 kg each.<sup>3</sup> Figure 1(a) sketches a cross section through the ship, indicating the two decks, the water line, and the keel. Galleons have a keel that is constructed from a single oak beam and is thus relatively small (in contrast to keels on modern sailboats), as is confirmed in the movie by shots of the bottom of the ship as it rolls over. Figure 1(b) indicates several important points and distances related to the stability of the galleon. The ship is assumed to be symmetric in shape and mass distribution from left to right (port to starboard) and is thus evenly divided by the vertical line on the figure. An inward projection of the surrounding water level would intersect that vertical line at the point of flotation O. The center of gravity (or mass) G of the ship is located above O because the draft of galleons is small and there is a lot of weight above the water due to cannons (18 on the lower deck and 14 on the upper deck), masts (three main ones), and the stern castle. The center of buoyancy B is the center of mass of a volume of water that would fill the submerged portion of the ship, shaded in blue in Fig. 1(b). The definition and critical importance of the metacenter and its height  $h$  above G will be discussed below.

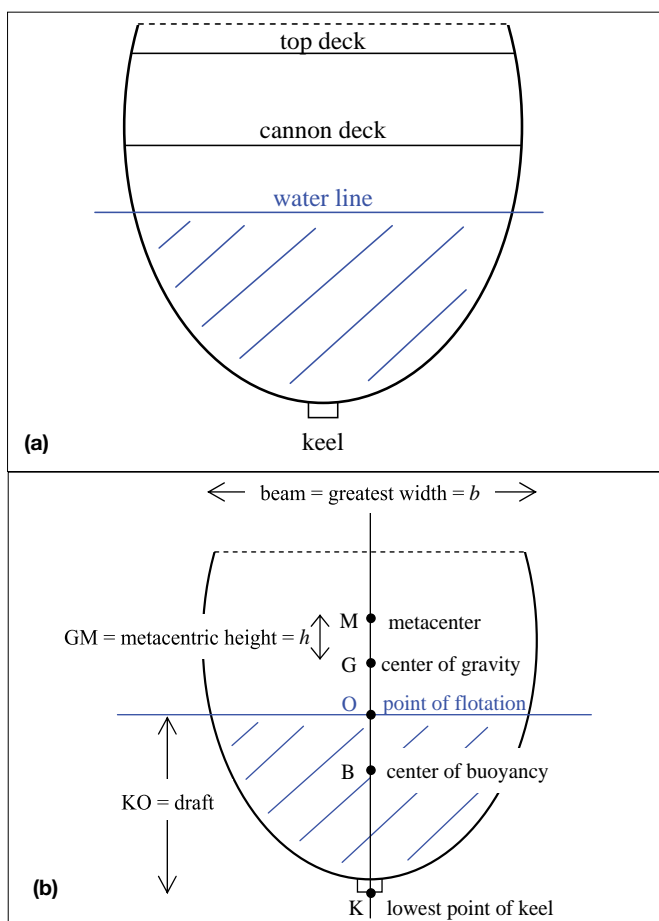
To analyze the rotational stability of the ship, we estimate its moment of inertia as though it were a solid cylinder of mass  $M$  and diameter  $b$ ,

$$I \approx \frac{1}{2} M \left( \frac{1}{2} b \right)^2 = 8.1 \times 10^6 \text{ kg} \cdot \text{m}^2. \quad (1)$$

Although it might initially seem preferable to use a semi-cylinder rather than a whole cylinder to estimate  $I$ , keep in mind that the masts are long and heavy, thereby adding moment of inertia to the top third of the sketch in Fig. 1(a). Also note that there is cargo and ballast in the interior of the ship, so that it is better to model it as a solid than as a hollow cylinder. (In fact, a purpose of the inward slope along the rising hull of galleons, called the “tumblehome bulge,” is to move the top cannons’ weight closer to the centerline so as to improve stability.<sup>4</sup>)

The radius of gyration is related to the moment of inertia by

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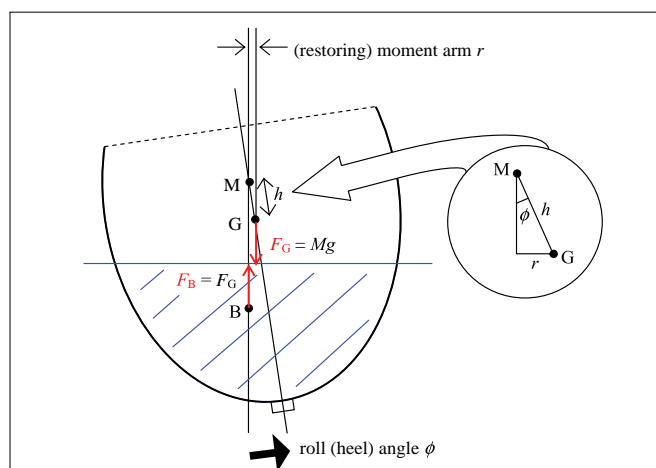
**Fig. 1. (a) Cross section about midway along the length of the ship, sketching the profile of the hull, and approximate location of the decks, water line, and keel. (b) Key points and lengths of interest on the cross section for the analysis of the stability of the ship.**

$$R \equiv \sqrt{\frac{I}{M}} = 4.0 \text{ m.} \quad (2)$$

As is derived below, the period at which a ship naturally rolls back and forth through small angles is related to this radius and the metacentric height by

$$T = 2\pi R / \sqrt{gh_0}, \quad (3)$$

where  $g = 9.8 \text{ m/s}^2$  at sea level. (The zero subscript on  $h$  indicates that it is the small-angle metacentric height.) Shipbuilders define a dimensionless rolling (Kempf) factor of  $T\sqrt{g/b}$ . It is known<sup>5</sup> that a rolling factor of less than 8 gives rise to a “stiff” ship, with sudden rolls that are unpleasant for crew and can cause cargo to shift, while rolling factors above 14 lead to a “tender” ship that can heel dangerously far to one side before righting itself. We can thus assume that the *Black Pearl* has a rolling factor of about 10, so that for small roll angles its metacentric height is  $h_0 = 0.56 \text{ m}$  (or more generally about 5% of the beam of a ship) and its period of oscillations is  $T \approx 11 \text{ s}$ . That period rough-



**Fig. 2. Forces and distances relevant to calculation of the restoring torque on a rolling ship. The inset circle zooms in to show that  $r = h \sin \phi$ .**

ly accords with the authors’ experience aboard tall ships at tourist attractions.

Now consider Newton’s second law for rotations,  $\tau = I\alpha$ . Ignoring viscous drag for the time being, the torque  $\tau$  on the ship is due to a force couple, as sketched in Fig. 2: the gravitational force  $F_G = Mg$  acting vertically downward through the center of gravity  $G$  and the buoyant force  $F_B$ , which according to Archimedes’ principle acts vertically upward through the center of buoyancy with a magnitude equal to the weight of the displaced water shaded in blue. Because the ship is floating, these two forces must be equal and opposite. The diagram in Fig. 2 shows the ship at some instant when it has rolled (“heeled”) through some angle  $\phi$ . The boat’s center of gravity remains on the centerline (passing through the keel). But the surface of the water is no longer perpendicular to that centerline. Instead, as the ship rolls counterclockwise (defining the conventional positive direction for  $\phi$ ), the center of buoyancy must move leftward, as shown in the figure. The metacenter  $M$  is defined as the point of intersection between the lines of action of the buoyant force when  $\phi$  is slightly varied. (For small angles,  $M$  will therefore be located on the ship’s centerline.) Remembering that for a couple, the torque has the same value when calculated about any origin, it is convenient to calculate  $\tau$  about  $M$  so that the buoyant force drops out and one simply obtains  $\tau = -Mgr$ , where the “righting moment arm” is equal to the horizontal projection of the metacentric height,  $r = h \sin \phi$ , and the minus sign accounts for the fact that the torque is restoring (acting to bring  $\phi$  back toward zero). Since the angular acceleration  $\alpha$  is equal to the second derivative of  $\phi$ , Newton’s second law becomes

$$-Mgh \sin \phi = MR^2 \frac{d^2 \phi}{dt^2} \quad (4)$$

using Eq. (2). For small angles (in radians), we can replace  $h$  by  $h_0$  and  $\sin \phi$  by  $\phi$  to get

$$\frac{d^2\phi}{dt^2} = -\Omega^2\phi, \quad \text{where} \quad \Omega \equiv \frac{\sqrt{gh_0}}{R}. \quad (5)$$

(To avoid confusion in the subsequent analysis, we reserve the symbol  $\omega$  to mean  $d\phi/dt$ .) Equation (5) describes the simple harmonic motion of a physical pendulum where  $h_0$  is the distance between the pivot point and the center of gravity. Equation (3) follows from this result because  $\Omega = 2\pi/T$ .

For large rolling angles, however,  $h$  does not remain constant at value  $h_0$  because the metacenter  $M$  shifts position. As one can see from Fig. 2, the only way that  $M$  could remain fixed in location for arbitrary  $\phi$  is if the hull of the ship were exactly cylindrical in shape (and no cargo—including hanging loads such as booms and “free” surfaces of liquids such as drinking water<sup>5</sup>—shifted during rolling), in which case the metacenter would coincide with the hull’s center of curvature.<sup>6</sup> In practical terms, if  $h$  were constant then  $r$  would be linearly proportional to  $\sin \phi$  and hence there would be a restoring torque even if the ship were nearly completely inverted (since  $\sin \phi > 0$  for  $0^\circ < \phi < 180^\circ$ , cf. the red curve in Fig. 3). Although small sailboats (with weighted keels) can in fact right themselves when the mast is underwater, tall ships cannot. We assume that the righting arm  $r$  of the *Black Pearl* falls to zero at a maximum heeling angle of about  $\phi_{\max} = 60^\circ$  (by which point the decks and hold can flood) by modeling  $h$  as decreasing linearly with angle, so that

$$h = h_0 \frac{\phi_{\max} - \phi}{\phi_{\max}}. \quad (6)$$

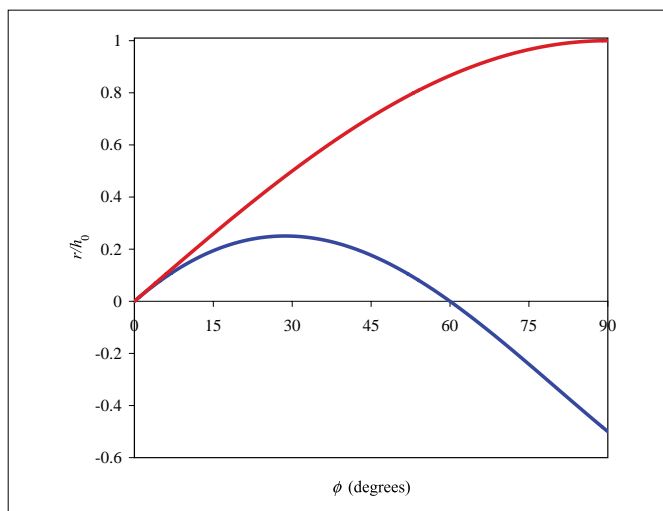
In that case,  $r/h_0$  versus  $\phi$  (called the “curve of statical stability”) is as plotted in blue in Fig. 3, resembling those of the tall ships *HMS Captain* and *HMS Monarch* from Fig. 6.23 of Ref. 5.

## Numerical integration of the roll as a function of time

Let’s now add the next level of detail to the stability analysis presented above by allowing the 16 runners on the top deck (with total mass  $m_r = 16 \times 88 \text{ kg} = 1400 \text{ kg}$ ) to travel back and forth between the two side rails. One can see in the movie that they all remain pretty much together as they run, so that we can model the whole group of them as a single point particle. The difference in vertical height between the top (and cannon) deck and the metacenter in Fig. 1 is unknown but probably small and so for simplicity will be neglected in this paper. Using body axes fixed to the ship with origin at the metacenter, it follows that only the horizontal position  $x$  (with the positive direction pointing to the right in Figs. 1 and 2) of the runners is relevant. Consequently, Newton’s second law for rotations now becomes

$$\tau_s + \tau_r = (I_s + I_r) \frac{d^2\phi}{dt^2}, \quad (7)$$

where the torque due to the ship (without the runners) is  $\tau_s = -m_s g r$  with net mass  $m_s = M - m_r$  and with righting



**Fig. 3.** Moment arm of the net torque (normalized to the small-angle metacentric height) as a function of heeling angle in the static limit where viscous drag can be ignored. The upper (red) curve would be the result if the metacenter remained fixed in position as the ship rolled. The lower (blue) curve more realistically models the metacentric height as linearly falling to zero at  $60^\circ$ , so that the restoring torque peaks at a roll angle near  $30^\circ$  rather than at  $90^\circ$ .

moment arm

$$r = h_0 \frac{\phi_{\max} - \phi}{\phi_{\max}} \sin \phi \quad (8)$$

from Eq. (6), and where the ship’s moment of inertia is  $I_s = m_s R^2$ . The torque due to the lateral displacement of the runners is  $\tau_r = -m_r g x \cos \phi$  (since  $x \cos \phi$  is the component of the crew members’ displacement parallel to the waterline, and where the minus sign indicates that a rightward  $x$  would produce a clockwise torque, as one can see from Fig. 2) and their moment of inertia is  $I_r = m_r x^2$ .

The ship starts at rest in an unheeled orientation, so that the initial conditions are  $x_0 = 0$ ,  $\phi_0 = 0$ , and  $\omega_0 = 0$ . For simplicity, we assume the crew runs back and forth with constant speed  $v$ , turning around whenever they reach the deck railings located at approximately  $x = \pm b/2$ . A logical choice of running speed that the crew could intuitively adopt would be to match the ship’s small-angle rolls, so that  $v = 2b/T = 2.1 \text{ m/s}$  using Eq. (3), not an unreasonable speed even for an out-of-shape pirate. The numerical integration is performed in a spreadsheet using timesteps of  $\Delta t = T/100$ . In step  $i$  (running from 1 up to about 1100, to span two minutes of motion), the current time is  $t = i\Delta t$  and the preceding time is  $t_{\text{prev}} = (i-1)\Delta t$ . The runners are currently located at  $x = x_{\text{prev}} \pm v\Delta t$ . Either a conditional test can be inserted in the code to reverse the  $\pm$  sign in this expression whenever a railing is reached, or more simply one can manually reverse the sign in the spreadsheet in the next step after that condition occurs. Finally, Eq. (7) is integrated in two steps using the Euler-Cromer method.<sup>7</sup> First the angular speed at the current time is obtained as

$$\omega = \omega_{\text{prev}} + \alpha_{\text{prev}}\Delta t, \quad (9)$$

where the angular acceleration at the preceding time is  $\alpha_{\text{prev}} = \tau_{\text{prev}}/I_{\text{prev}}$ , with net torque  $\tau = \tau_s + \tau_r$  and moment of inertia  $I = I_s + I_r$  from Eq. (7). For the given initial conditions, we can compute all needed quantities at  $t = 0$  to start the time iteration. Then the current angular speed is used to compute the current angular position according to

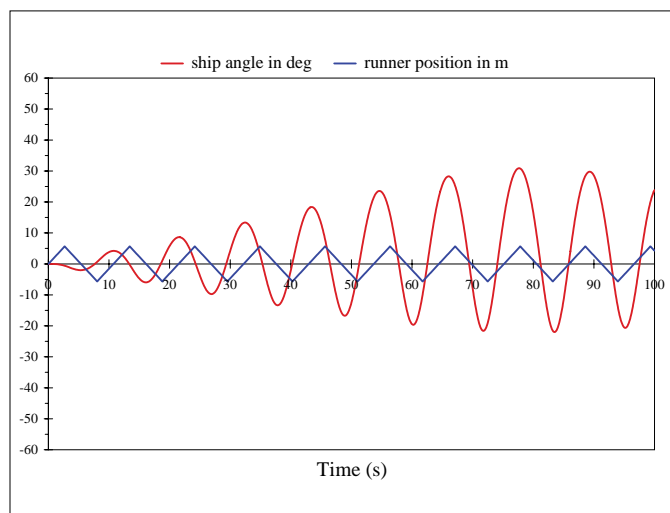
$$\phi = \phi_{\text{prev}} + \omega\Delta t. \quad (10)$$

The results are plotted in Fig. 4. (In this and the following three graphs, the vertical axis extends to the  $\pm 60^\circ$  angle that the ship needs to attain if the restoring torque is to fall to zero so that the *Black Pearl* can roll over.) The crew's displacement describes a triangle wave (in the ship's frame of reference) as they run at constant speed between the two railings located at  $\pm 5.65$  m. The angular amplitude of the ship increases during the first minute after the crew begins running, but it reaches a maximum of about  $30^\circ$  and thereafter begins to decrease. The reason for this behavior is that the runners get out of synchronicity with the ship. They are running to match the small-angle behavior of the boat, but just as for a pendulum, the period of oscillation increases at large angles.

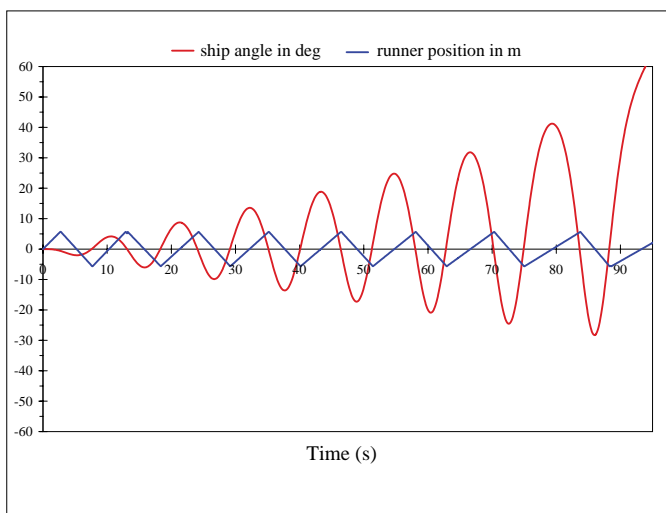
To maintain maximum coupling between the oscillations of the crew and of the ship, the runners need to adjust their speed to maintain a  $90^\circ$  phase shift between them and the boat. (Recall that the amplitude of forced oscillations is a maximum when a driver and oscillator are  $90^\circ$  out of phase with each other. In the present context, such a phase shift guarantees that the runners will be to the *left* of the centerline [negative  $x$ ] whenever the *left* edge of the ship is descending

[positive  $\omega$ ], and likewise they will be on the right side for clockwise rolling. Thus the runners cross the centerline when the ship has maximum tilt, and they hit the rails when the ship has zero tilt, as one can see in Fig. 5.) To do so, we from now on assume that they can choose a different constant speed in each half-cycle (between one deck railing and the other) after stopping and turning around. Specifically, we modified the spreadsheet so that whenever the crew reaches a rail, the ship is always within  $1^\circ$  of  $\phi = 0$ . To accomplish that feat, we simply varied the speed  $v$  (typically by a few tenths of meters per second) by trial and error in each half-cycle (after the first one, which remained at 2.1 m/s). One might argue that the crew has no way to predict their required speed by trial and error in this fashion. But in the first place, the crew gets visceral feedback that they can use to correct errors in their running pattern, just as a person pushing a child on a swing through large angles somehow manages to figure out how to adjust their pushes without knowing any physics or having access to a fast computer. In the second place, real runners would not be constrained to run at constant speed but could dynamically adjust their speed “on the fly,” something that could probably be coded into our spreadsheet but only with considerable extra work. Our simple model has the benefit that it emphasizes for students the important idea of resonance, because as Fig. 5 demonstrates, the crew now succeeds in overturning the ship.

But once one realizes that undamped resonance is responsible for this success, one also recognizes that the crew could succeed even if only one small person ran back and forth, which is clearly unrealistic! Any sensible model must therefore include damping. Consider hull drag in the water. The nature of this fluid resistance force depends on the Reynolds number  $N = vD/\nu$ . Here  $v$  is the lateral speed of the hull rela-



**Fig. 4.** Results of a numerical integration ignoring viscous drag in which the crew members run back and forth at a fixed speed matching the small-angle rolls of the ship, while the cannons remain fixed in position. The *Black Pearl* never surpasses  $30^\circ$  of roll.



**Fig. 5.** To improve upon the results of Fig. 4, the crew adjusts their speed in each half-cycle to maintain a  $90^\circ$  phase shift relative to the ship. Note the asymmetry in the speeds with which they run in the positive and negative directions, as they “pump” the ship preferentially toward positive angles until it rolls over. The symmetry was “broken” by the direction the crew originally chose to run and “amplified” by the fact that rollover occurs after only a small number of half-cycles (specifically 16).

tive to the water and it depends on the ship's angular speed  $\omega$  as  $v = \omega d$ . Rotational drag should be most pronounced at the keel, which resists lateral displacements. The distance from the keel K to the flotation point O (which specifies the ship's rotational axis<sup>6</sup>) is equal to the draft,  $d = 3.8$  m. The kinematic viscosity of seawater<sup>8</sup> at 15°C is  $\nu = 1.16 \times 10^{-6}$  m<sup>2</sup>/s. Galleons have a small keel<sup>9</sup> with a height of only about  $D = 8$  in = 0.2 m. In the simulations in Figs. 4 and 5,  $\omega$  has a typical value of about 0.1 rad/s. Consequently, the Reynolds number for the keel is about  $N = 65,000$ . At such a large Reynolds number, the force and hence the torque due to drag is quadratic in the speed and we can calculate it as

$$\tau_D = -\beta\omega|\omega|, \text{ where } \beta = \frac{1}{2}\rho C A d^3. \quad (11)$$

Here  $\rho = 1020$  kg/m<sup>3</sup> is the density of seawater,  $C = 2$  is the drag coefficient of a long and narrow plate or rectangular cylinder<sup>8</sup> (such as the keel seen from the side), and  $A = DL$  is the area of the keel (where the ship has length  $L = 55$  m). Two powers of  $d$  in Eq. (11) arise to convert between  $\omega$  and  $v$ , and the remaining power (to give  $d^3$  in all) converts force into torque. Equation (11) is written with a minus sign and absolute value for ease of numerical computation where  $\omega$  is the signed angular velocity of the ship.

Substituting the above values, one finds  $\beta = 6 \times 10^5$  N·m·s<sup>2</sup> for the keel. It is reasonable to assume that a similar value will apply for the rudder plus hull (which must push some water out of the way since it is not perfectly round) and again for the air resistance of the sails, stern castle, and masts.<sup>10</sup> (Lamentably, in the movie the crew does not think to furl the large lateen sails.) Thus a three times larger value of  $\beta = 2 \times 10^6$  N·m·s<sup>2</sup> is adopted. (Using a larger value of  $\beta$  in this manner errs on the side of caution.) Equation (7) is modified to read

$$\tau_s + \tau_r + \tau_D = (I_s + I_r) \frac{d^2\phi}{dt^2}. \quad (12)$$

Now even when the crew runs in the manner of Fig. 5 to maintain the ideal 90° phase shift, they simply do not have enough mass to roll the ship over. As graphed in Fig. 6, they can only drive its angular amplitude up to 20°.

Finally we include the effect of loosening the cannons from their restraints on the gun deck. They are so heavy that if one lined all 18 of them along one wall of the ship, it would capsize immediately. To see this, note that the maximum restoring torque of the *Black Pearl* is about  $Mgh_0/4 = 7 \times 10^5$  N·m at an angle of 30°. At that same angle, cannons of total mass  $m = 18 \times 2000$  kg could exert a toppling torque of  $mg(b/2) \cos 30^\circ = 1.7 \times 10^6$  N·m that is more than twice as large. However, in the movie there are several shots of the motion on the gun deck. Far from freely rolling across the deck, the cannons are seen to overturn some of their carriages, collide with the support columns and deck stairs, and run into each other and the barrels that are also banging around. Consequently, let's suppose that about half the cannons are tangled up at any time, so that the effective oscillating mass of the cannons is reduced to

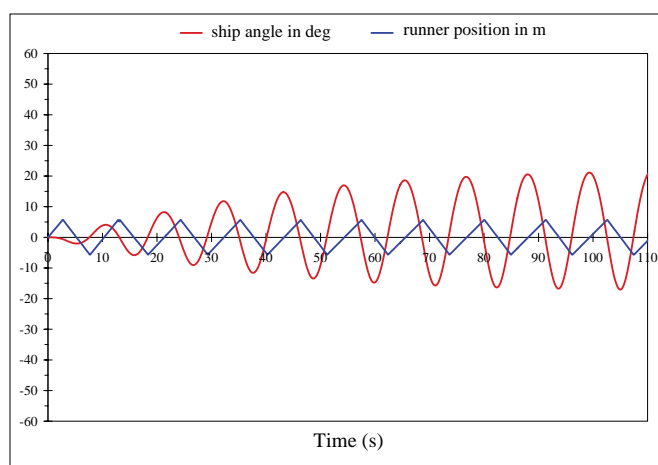


Fig. 6. By including fluid drag in the model, the crew alone are not able to roll the *Black Pearl* over, even for ideal 90° forcing.

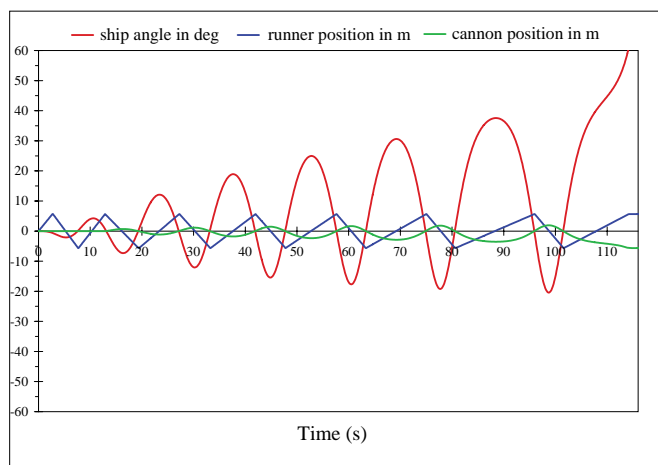


Fig. 7. The successful final model that includes moving cannons, viscous drag, and crew running at a speed that is readjusted each half-cycle to maintain a 90° phase shift relative to the roll of the ship. To reproduce what occurs in the movie, the crew members grab onto the upper deck railing and stop running once the ship surpasses the 60° angle of no return. At the same instant, the cannons are piling up along the lower hull wall.

$m_c = 18,000$  kg. Denote the lateral position of this equivalent moving point mass by  $y$ , to distinguish it from the position  $x$  of the runners. Furthermore, unlike the crew members who can actively control their speed, the cannons passively follow the rocking of the ship. Specifically, let's assume their positions "adiabatically" adapt to the changing angle of the ship. That is, as the ship tilts from one angle to another, the interfering mass of cannons and barrels continually rearrange themselves into semi-stable arrangements such that the position  $y$  of the cannons is a function only of the angle  $\phi$  of the ship. A simple model takes  $y$  to vary linearly with  $\phi$ , and  $y$  reaches its maximum value (of  $b/2 = 5.65$  m) as the ship tilts to its maximum stable value (of  $\phi_{\max} = 60^\circ$ ),

$$y = -\frac{\phi b}{2\phi_{\max}}, \quad (13)$$



where the minus sign indicates that as the *Black Pearl* rolls counterclockwise, the cannons slide leftward (cf. Fig. 2).

So now Eq. (7) becomes

$$\tau_s + \tau_r + \tau_c + \tau_D = (I_s + I_r + I_c) \frac{d^2\phi}{dt^2}, \quad (14)$$

where the ship's bare mass is reduced to  $m_s = M - m_r - m_c$  in calculating  $\tau_s$  and  $I_s$ , the torque due to motion of the cannons is  $\tau_c = -m_c g y \cos \phi$ , and their effective moment of inertia is  $I_c = m_c y^2$ . The results of the numerical integration are plotted in Fig. 7. We see that the ship rolls over at about the two-minute mark. Amazingly, if one times the movie starting from when the crew begins running, and ending when the *Black Pearl* overturns, about two minutes do in fact elapse! Although this exact agreement must be coincidental, given the number of simplifications in our analysis, it is nevertheless encouraging.

## Concluding remarks

The preceding analysis suggests that the physics in this movie clip is realistically portrayed, making it a good case study for students. Rolling is apparently common for galleons such as the *Black Pearl*, as those ships are known<sup>4,11</sup> to be unstable because of their narrow beam, short keel, and top-heaviness. One is reminded of the Swedish galleon the *Vasa*, which is of a similar size and vintage as the *San Juan Bautista*, upon which the present analysis is based. The *Vasa* heeled and sank after barely beginning its maiden voyage. There is a Swedish museum housing the salvaged wreck, which distributes an illustrated 24-page brochure about the ship.<sup>12</sup> In it, one reads the remarkable words “a stability test had been performed prior to the maiden voyage. Thirty men had run back and forth across the *Vasa*'s deck when she was moored at the quay. The men had to stop after three runs, well before the test could be completed—otherwise, the ship would have capsized.” This statement gives strong credence to our results.

Because of the form we have adopted for the stability curve (cf. the blue graph in Fig. 3), our model necessarily implies that if the ship rolls past 60°, it will continue to roll all the way around to 180°. One may reasonably question the accuracy of this feature of the analysis, but it is difficult to confirm it based on our theory, for several reasons:

- cargo and ballast will begin shifting around to the walls and ceilings of the holds in ways that depend on their architecture;
- one or more compartments of the ship may begin to flood (fortunately in the movie the crew seems to have remembered to close the gunport flaps after loosing the cannons);
- viscous drag on the sails would increase considerably as they cross from air to water.

Perhaps it might be possible to build and test the behavior of a tall ship model as its masts are inverted, although that might require a fairly detailed model to convincingly address

the preceding bullet points. Certainly it would be interesting to know if any large historical sailing vessels have ever fully rolled over, say, during a violent storm.

Teachers interested in presenting this movie clip in class can easily do so if they have or rent a copy of the film on DVD. Simply go to the Scene Selection menu and choose item 9: “Rock and Roll.”

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10. Use the formula in Eq. (11) for air resistance with  $\rho = 1.2 \text{ kg/m}^3$  (for moist air at sea level) and  $C = 1.2$  for a square plate at high Reynolds number. Many of the sails are oriented perpendicular to the length of the ship and thus do not contribute to the rolling drag. From the movie we estimate a net lateral area for contributing surfaces of about  $A = 200 \text{ m}^2$  at a mean height above the water of about  $d = 15 \text{ m}$  so that  $\beta = 5 \times 10^5 \text{ N}\cdot\text{m}\cdot\text{s}^2$ , comparable to what we calculated for the keel.
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